Three-dimensional flow in a cylinder with a stress-free sidewall

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Received 5 June 2005; received in revised form 4 February 2006; accepted 19 August 2006

Communicated by T. Kambe

Abstract

Stokes flow in a cylindrical column of fluid, with a stress-free cylindrical sidewall, is considered. The motion is assumed to be generated by the linear, uniform motion of either or both of the flat endwalls. The field is obtained by a vector eigenfunction expansion procedure. If the field is assumed to have a \( \theta \)- and \( z \)-dependence of the type \( \exp(i\theta + kz) \), \( k \) has to satisfy the equation \( (1 - k^2) J_1^2 + \tilde{J}_1[2J_2^2 - \tilde{J}_1(2\tilde{J}_1 + (1 + k^2)J_1)] = 0 \), where \( \tilde{J}_1 = kJ_0 - J_1 \) and the argument of each Bessel function is \( k \). This equation admits, unlike in the plane case and with important consequences, not just a real sequence \( \{ \lambda_n \} \) of eigenvalues but also a complex one \( \{ \mu_n \} \). Using a least squares procedure to satisfy the boundary conditions on the top and bottom walls, the three-dimensional velocity field in the column is determined for various values of column height \( h \) and wall speed ratio \( S \). Detailed computations show that there are strong effects of both the stress-free boundary and three-dimensionality. The principal effect of the former is to permit motion on that boundary leading to large azimuthal motions and of the latter, unlike in the plane flow, to multiple primary eddies when \( h \) is sufficiently large. A number of new eddy structures are also found, which demonstrate that three-dimensionality often leads to the elimination of compactness found in plane flows. It is finally shown that the flow fields exhibit interesting bifurcations as \( S \) and \( h \) are varied.

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Keywords: Three-dimensional Stokes flow; Stress-free boundaries; Eddy structure; Meniscus roll coating; Vector eigenfunction expansions; Bifurcations

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doi:10.1016/j.fluid.2006.08.010