Volume integrals of the first and third invariants of the velocity gradient tensor in incompressible flows

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Volume integrals of the second and third invariants, i.e. QA and RA, respectively, of the velocity gradient tensor $A_{ij}$ over an incompressible flow domain are shown to vanish for certain combinations of boundary conditions used in a large variety of direct numerical simulations of turbulent flows. For these turbulent flows, the dissipation of total kinetic energy is directly proportional to the total enstrophy. Betchov [J. Fluid Mech. 1 (1956) 497] showed that for homogeneous flows an increase in mean enstrophy implies that it is more likely that the intermediate rate-of-strain is positive rather than negative. This paper shows that using the framework of the invariants of the velocity gradient tensor and a volume integral formulation an analogous implication can be derived for more general classes of flows. Specifically, in addition to the case of homogeneous flows, this paper shows that in the cases of unbounded inhomogeneous flows (i.e. free shear flows) and for wall-bounded semi-infinite domain flows with zero tangential pressure gradient an increase in total QW, the second invariant of the rate of rotation tensor, implies that it is more likely that the intermediate rate-of-strain is positive rather than negative.

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