Eulerian mean current and Stokes drift under non-breaking waves on a perfect fluid over a plane beach

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The flow associated with small amplitude non-breaking gravity waves on a perfect fluid in a wedge shaped domain is described by a second-order perturbation analysis. Retention of vertical accelerations to the leading order yields a (non-hydrostatic) model which is applicable to arbitrary slopes. A Wronskian $W$ is written of the two fundamental solutions. It is shown that, for beaches of slope angle $\alpha = \pi/2m$, $m \leq 2$, $RW$ is represented by a Taylor expansion in $R$, the distance from the fixed shoreline. An absolutely convergent integral expression is derived for the steady Eulerian velocity potential in a situation which is applicable to waves which are both standing and progressing in the far field. A derivation is used to compute induced steady bottom current, for a variety of slopes. The strength of this current is everywhere proportional to $(1 - Q^2)$, where $Q$ is the reflection coefficient for the first-order wave motion. The computed current is found to have a reversal point typically at a few wavelengths from the shoreline. Following an application of the convolution theorem, an expression is derived which may be applied throughout the depth. This enables “stream function” contouring for the second-order Eulerian current from which an “updraught” is indicated above the reversal point. An expression for the Stokes drift is also derived, computed and mapped. It is found that “stream curves” for the resulting mass transport velocity are essentially horizontal except very close to the bed. Both Eulerian current and Stokes drift vanish identically in the case of perfect reflection. It is found, for gentle slopes, that values of $W$ on the equilibrium surface $\Sigma$ may be well approximated by $kD^2d/k^\infty$; where $Dd$ is the Airy theory shoaling coefficient and $k$ and $k^\infty$ are the local and deep-water wave numbers respectively. This approximation permits the current to be expressed purely in terms of Airy theory parameters and is therefore computable without the need for computing first-order potential functions. Computations show excellent agreement with those obtained from the integral of the more exact theory. It is further established that the steady potential on $\Sigma$ may be computed by a Cauchy principal value integral thus facilitating computation throughout the water column under the new approximation. A mapping of “equipotentials” is seen to be entirely consistent with the work under the more exact theory.